**Trabajo Práctico N° 6:**

**Integrales.**

**Ejercicio 1.**

**(a)** *Si = 37 y = 16, encontrar el valor de .*

= +

= 2 -

= 2 \* 37 - \* 16

= 74 - 4

= 70.

**(b)** *Si = 12 y = 3, hallar el valor de .*

= -

= 12 - 3

= 9.

**(c)** *Si = 3 y = 7, determinar el valor de .*

= -

= 7 - 3

= 4.

**Ejercicio 2.**

*Calcular las siguientes integrales utilizando las propiedades y, en caso de ser posible, usando la regla de Barrow.*

**(a)** *.*

= +

= 2 -

= 2 - x

= - [3 - (-2)]

= [ - ] - (3 + 2)

= (9 - 4) - 5

= 5 - 5

= 0.

**(b)** *.*

= + +

= + 2 + 8

= + 2 + 8x

= + + 8x + C.

**(c)** *.*

= +

= -cos x +

= -(cos 2 - cos 0) + [ - ]

= -(1 - 1) + (4 - 0)

= -0 + 4

= 2.

**(d)** *.*

= +

= 2 + 3

= 2 + 3

= 2 ( - ) + ( - )

= 2 ( - 1) + (1024 - 0)

= 2 - 2 + \* 1024

= 2 - 2 +

= .

**(e)** *.*

= +

= 3 + 2

= 3 ln + 2 + C.

**(f)** *.*

= + +

= sen x - cos x + 2

= sen x - cos x + 2

= sen x - cos x + + C.

**(g)** *.*

= + +

= + 2 + 8

= [ - ] + 2 + 8 x

= [1 - (-125)] + [ - ] + 8 [1 - (-5)]

= (1 + 125) + (1 - 25) + 8 (1 + 5)

= \* 126 - 24 + 8 \* 6

= - 24 + 48

= 66.

**(h)** *.*

= + + +

= - + 3 -

= - + 3 - sen x

= - + 3 - sen x + C.

**Ejercicio 3.**

*Calcular las siguientes integrales utilizando los métodos vistos.*

**(a)** *.*

= (\*)

=

= + C.

(\*) u= 3 + 5 + 8; du= (12 + 10x) dx.

**(b)** *.*

= x sen x - (\*)

= x sen x - (-cos x)

= x sen x + cos x + C.

(\*) u= x; du= dx; dv= cos x dx; v= sen x.

**(c)** *.*

= ln x - (\*)

= ln x -

= ln x -

= ln x -

= (ln x - ) + C.

(\*) u= ln x; du= dx; dv= dx; v= .

**(d)** *.*

= (\*)

= sen u

= sen 5x + C.

(\*) u= 5x; du= 5 dx.

**(e)** *.*

= (\*)

= ln

= ln + C.

(\*) u= + 2x; du= ( + 2) dx.

**(f)** *.*

(\*) u= ; du= dx.

= (\*)

=

= +

= +

= +

= 2 [ + ] + C.

(\*) u= x - 1; du= dx.

**(g)** *.*

= (\*)

=

=

= 2

= 2

= 2 ( - )

= 2 ( - )

= 2 (3 - 1)

= 2 \* 2

= 4.

(\*) u= x + 1; du= dx.

**(h)** *.*

= x (-cos x) - (\*)

= -x cos x +

= -(2 cos 2 - 0 cos 0) + sen x

= -(2 \* 1 - 0 \* 1) + (sen 2 - sen 0)

= -(2 - 0) + (0 - 0)

= -2 + 0

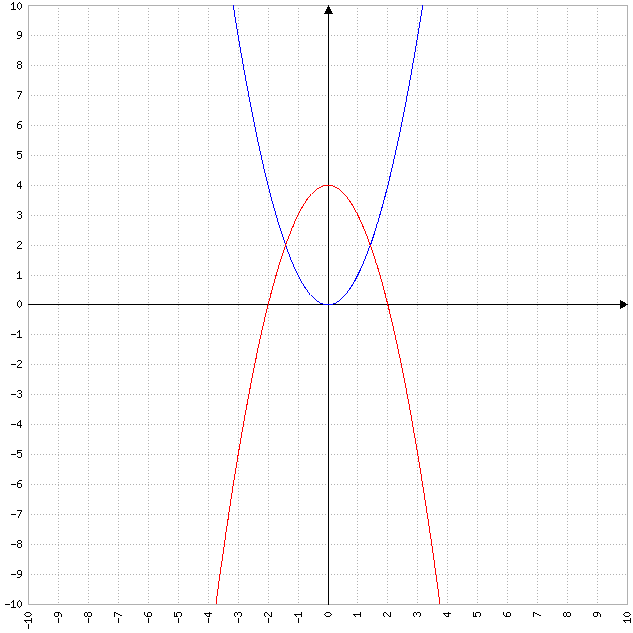
= -2.

(\*) u= x; du= dx; dv= sen x dx; v= -cos x.

**Ejercicio 4.**

*Hallar el área comprendida entre las gráficas de las siguientes pares de funciones:*

**(a)** *f (x)= y g (x)= - + 4.*



f (x)= g (x)

= - + 4

+ = 4

2= 4

=

= 2

=

=

x= .

|  |  |
| --- | --- |
| **Intervalo** | **(-, )** |
| VP | 0 |
| f (x) | 0 |
| g (x) | 4 |

A=

A=

A=

A=

A= +

A= -2 + 4

A= -2 + 4x

A= [ - ] + 4 [ - (-)]

A= [ + ] + 4 ( + )

A= \* 2 + 4 \* 2

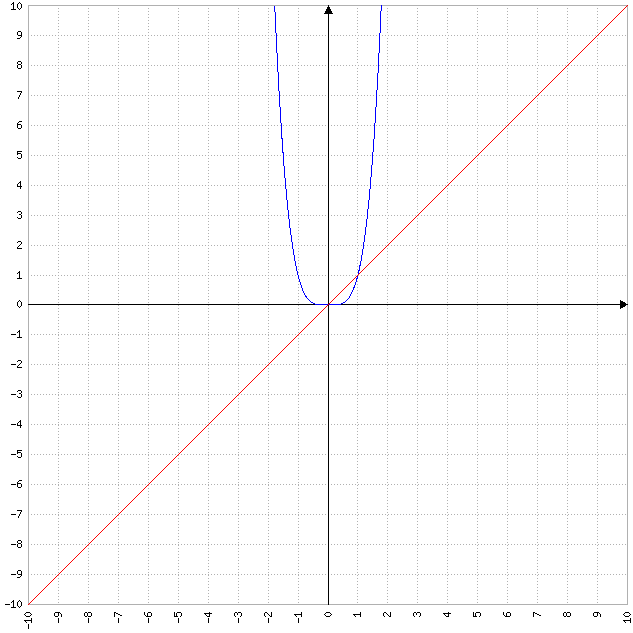
A= + 8

A= \* 2 + 8

A= + 8

A= .

**(b)** *f (x)= y g (x)= x.*



f (x)= g (x)

= x

- x= 0

(x - 1)= 0.

= 0; = 1.

|  |  |
| --- | --- |
| **Intervalo** | **(0, 1)** |
| VP |  |
| f (x) |  |
| g (x) |  |

A=

A=

A= +

A= -

A= ( - ) -

A= (1 - 0) - ( - )

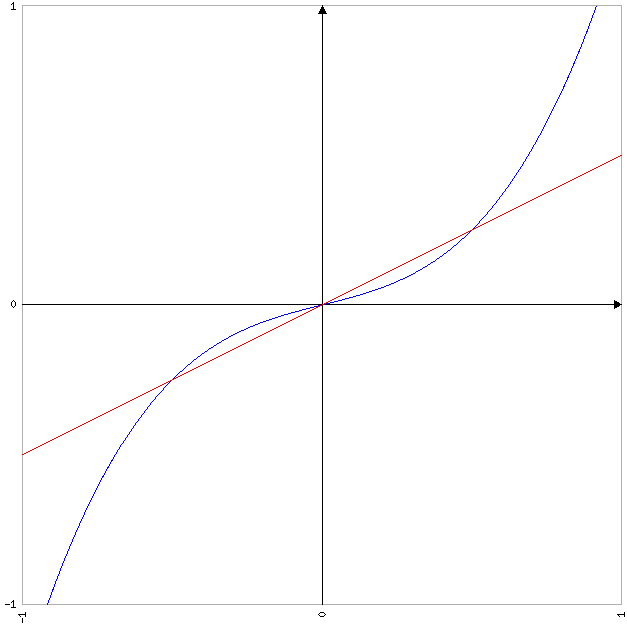
A= \* 1 - (1 - 0)

A= - \* 1

A= -

A= .

**(c)** *f (x)= + x y g (x)= x.*



f (x)= g (x)

+ x= x

+ x - x= 0

-  x= 0

x ( - )= 0.

= 0; = ; = .

|  |  |  |
| --- | --- | --- |
| **Intervalo** | **(, 0)** | **(0, )** |
| VP |  |  |
| f (x) |  |  |
| g (x) |  |  |

A= +

A= +

A= +

A= +

A= + + +

A= - - +

A= - - +

A= [ - ] - [ - ] - [ - ] + [ - ]

A= (0 - ) - (0 - ) - ( - 0) + ( - 0)

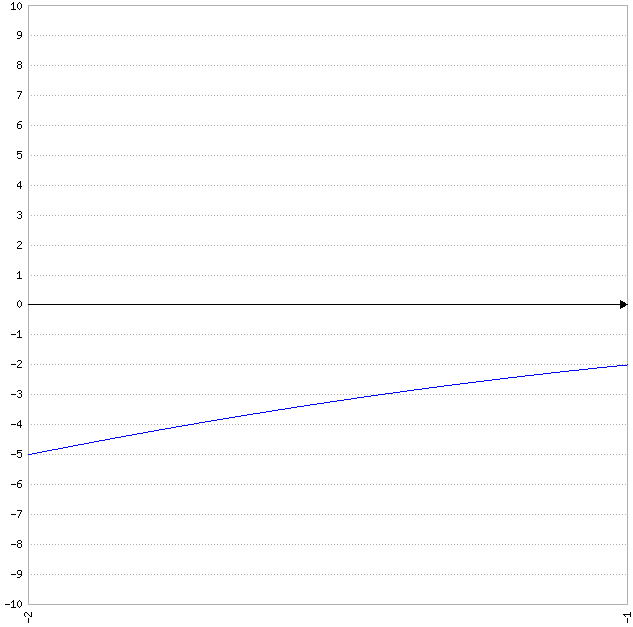
A= () - () - +

A= + - +

A= .

**Ejercicio 5.**

*Calcular el área de la región comprendida entre el eje x y el gráfico de la función f (x)= - - 1 entre -2 x -1.*



f (x)= 0

- - 1= 0

-1.

|  |  |
| --- | --- |
| **Intervalo** | **(-2, -1)** |
| VP |  |
| f (x) | 0 |

A= -

A= -( + )

A= -(- - )

A= -( - x )

A= -{ [ - ] - [-1 - (-2)]}

A= -{ [-1 - (-8)] - (-1 + 2)}

A= -[ (-1 + 8) - 1]

A= -( \* 7 - 1)

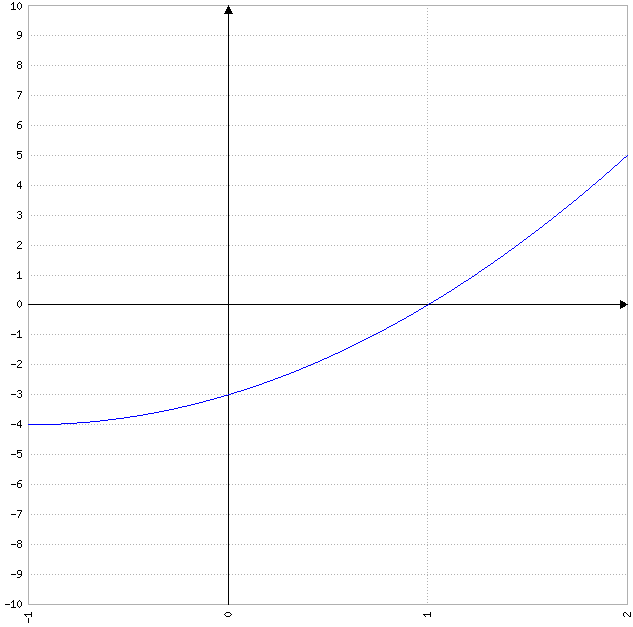
A= -( - 1)

A= -()

A= .

**Ejercicio 6.**

*Calcular el área de la región comprendida entre el eje x y el gráfico de la función f (x)= + 2x - 3 entre -1 x 2.*



f (x)= 0

+ 2x - 3= 0.

, =

, =

, =

, =

= = = 1.

= = = -3.

|  |  |  |
| --- | --- | --- |
| **Intervalo** | **(-1, 1)** | **(1, 2)** |
| VP | 0 |  |
| f (x) | 0 | 0 |

A= - +

A= -( + + ) + ( + + )

A= -( + 2 - 3 ) + ( + 2 - 3 )

A= -{ [ - ] + 2 - 3x } + [ ( - ) + 2 - 3x ]

A= -{ [1 - (-1)] + [ - ] - 3 [1 - (-1)]] + [ (8 - 1) + ( - ) - 3 (2 - 1)]

A= -[ (1 + 1) + (1 - 1) - 3 (1 + 1)] + [ \* 7 + (4 - 1) - 3 \* 1]

A= -( \* 2 + 0 - 3 \* 2) + ( + 3 - 3)

A= -( + 0 - 6) +

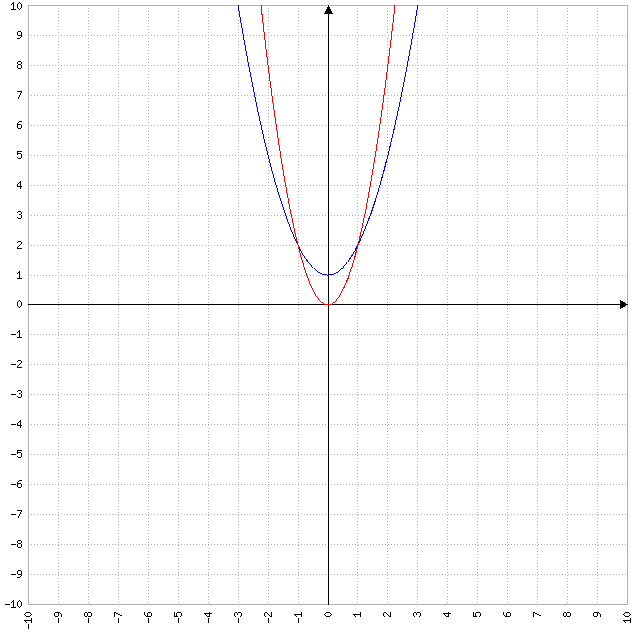
A= -() +

A= +

A= .

**Ejercicio 7.**

*Calcular el área de la región comprendida entre los gráficos de f (x)= + 1 y g (x)= 2 para 0 x 2.*



f (x)= g (x)

+ 1= 2

2 - = 1

= 1

=

= 1

x= 1.

|  |  |
| --- | --- |
| **Intervalo** | **(-1, 1)** |
| VP | 0 |
| f (x) | 1 |
| g (x) | 0 |

A=

A=

A=

A= +

A= - + x

A= + [1 - (-1)]

A= [ - ] + (1 + 1)

A= [1 - (-1)] + 2

A= (1 + 1) + 2

A= \* 2 + 2

A= + 2

A= .

**Ejercicio 8.**

*Hallar f (x) sabiendo que f´ (x)= x + y f (1)= 1.*

f (x)=

f (x)=

f (x)= +

f (x)= +

f (x)= +

f (x)= - + C.

f (1)= 1

\* - + C= 1

\* 1 - 1 + C= 1

- 1 + C= 1

+ C= 1

C= 1 +

C= .

f (x)= - + .

**Ejercicio 9.**

*Sabiendo que f´ (x)= 3 - 8x + 2 y, además, que f (3)= -4, hallar la función f (x).*

f (x)=

f (x)=

f (x)= + +

f (x)= 3 - 8 + 2

f (x)= 3 - 8 + 2x

f (x)= - 4 + 2x + C.

f (3)= -4

- 4 \* + 2 \* 3 + C= -4

27 - 4 \* 9 + 6 + C= -4

27 - 36 + 6 + C= -4

-3 + C= -4

C= -4 + 3

C= -1.

f (x)= - 4 + 2x - 1.

**Ejercicio 10.**

*Hallar todas las funciones cuya derivada es g´ (x)= cos x.*

g (x)=

g (x)=

g (x)= sen x - (\*)

g (x)= sen x - 2

g (x)= sen x - 2 [x (-cos x) - ] (\*\*)

g (x)= sen x - 2 (-x cos x + )

g (x)= sen x - 2 (-x cos x + sen x)

g (x)= sen x - 2x cos x - 2 sen x

g (x)= ( - 2) sen x - 2x cos x + C.

(\*) u= ; du= 2x dx; dv= cos x dx; v= sen x.

(\*\*) u= x; du= dx; dv= sen x; v= -cos x.

**Ejercicio 11.**

*Sea g´´ (x)= 2 - 4, g´ (1)= -2 y g (0)= 0, hallar la función g (x).*

g´ (x)=

g´ (x)=

g´ (x)= +

g´ (x)= 2 - 4

g´ (x)= 2 - 4

g´ (x)= -

g´ (x)= ( - ) + C.

g´ (1)= -2

( - ) + C= -2

(1 - 1) + C= -2

\* 0 + C= -2

0 + C= -2

C= -2.

g´ (x)= ( - ) - 2.

g (x)=

g (x)=

g (x)=

g (x)= + +

g (x)= - - 2

g (x)= - - 2x

g (x)= - - 2x + C.

g (0)= 0

- - 2 \* 0 + C= 0

\* 0 - \* 0 - 0 + C= 0

0 - 0 - 0 + C= 0

C= 0.

g (x)= - - 2x.